Reduction Formula 103

VIII form $\int_0^\infty x^n e^{-x}\,dx$

$$\Rightarrow \boxed{I_n = \int_0^\infty x^n e^{-x}\,dx = n! = n(n-1)(n-2)\ldots 3.2.1}$$

Ex Evaluate $\int_0^1 \left(\log \frac{1}{x}\right)^n dx$

Soln $I = \int_0^1 \left(\log\frac{1}{x}\right)^n dx$

Let $\log \frac{1}{x} = z \Rightarrow \frac{1}{x} = e^{z}$

$\Rightarrow x = e^{-z} \Rightarrow dx = -e^{-z}dz$

If $x \to 0 \Rightarrow z \to \infty$
If $x \to 1 \Rightarrow z \to 0$

$$\therefore I = \int_\infty^0 z^n (-e^{-z})\,dz = -\int_\infty^0 z^n e^{-z}\,dz$$

$$= \int_0^\infty z^n e^{-z}\,dz$$

$$= n! = n(n-1)(n-2)\ldots(3)(2)(1).$$

Ans

(IX) Form $\int \cos^m x \cos nx \, dx$

$$\Rightarrow \boxed{I_{m,n} = \frac{\cos^m x \sin nx}{m+n} + \frac{m}{m+n} I_{m-1,n-1}}$$

$$\& \boxed{I_{m,n} = \frac{\cos^m x \sin nx}{n-m} - \frac{m}{n-m} I_{m-1,n+1}}$$

$$\& \boxed{I_{m,n} = \frac{n\cos^m x \sin nx}{n^2-m^2} - \frac{m\cos^{m-1} x \sin x \cos nx}{n^2-m^2} - \frac{m(m-1)}{n^2-m^2} I_{m-2,n}}$$

(X) Form $\int_0^{\pi/2} \cos^m x \cos nx \, dx$

$$\Rightarrow \boxed{I_{m,n} = \frac{m}{m+n} I_{m-1,n-1}}$$

$$\& \boxed{I_{m,n} = \frac{m}{m-n} I_{m-1,n+1}}$$

$$\& \boxed{I_{m,n} = \frac{m(m-1)}{m^2-n^2} I_{m-2,n}}$$

Ex Evaluate $\int_0^{\pi/2} \cos^n x \cos nx\, dx$

Soln Since both $m, n = n$ i.e., m & n are equal

We use first formula

$$I_{m,n} = \int_0^{\pi/2} \cos^n x \cos nx\, dx = \frac{n}{n+n} I_{n-1,n-1}$$

$$= \frac{n}{2n} I_{n-1,n-1} = \frac{1}{2} I_{n-1,n-1} \quad \text{--- (1)}$$

Similarly For $I_{n-1,n-1}$

$$= \frac{n-1}{n-1+n-1} I_{n-2,n-2}$$

$$= \frac{n-1}{2(n-1)} I_{n-2,n-2}.$$

$$= \frac{1}{2} I_{n-2,n-2}$$

$$I_{n-2,n-2} = \frac{1}{2} I_{n-3,n-3}$$

$$I_{n-3,n-3} = \frac{1}{2} I_{n-4,n-4}$$ and so on

Substituting all the above in Eqn (1) we get

$$I_{n,n} = \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdots \text{to } n \text{ times}\right) \cdot I_{0,0}$$

$$= \left(\frac{1}{2}\right)^n I_{0,0} \quad \text{--- (2)}$$

$$I_{0,0} = \int_0^{\pi/2} \cos^0 x \cos 0x \, dx$$

$$= \int_0^{\pi/2} 1 \cdot \cos 0 \, dx = \int_0^{\pi/2} 1 \cdot 1 \cdot dx$$

$$= [x]_0^{\pi/2} = \frac{\pi}{2}$$

Substituting above value in eqn ② we get

$$I_{n,n} = \left(\frac{1}{2}\right)^n \cdot \frac{\pi}{2} = \frac{1}{2^n} \cdot \frac{\pi}{2}$$

$$\Rightarrow \int_0^{\pi/2} \cos^n x \cos nx \, dx = \frac{\pi}{2^{n+1}}$$ Ans